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Original Research

Investigating the Scaling Properties of Extreme Rainfall Depth Series in Oromia Regional State, Ethiopia

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Abstract

Depth Duration Frequency (DDF) relationships are currently constructed based on at site frequency analysis of rainfall data separately for different durations. These relationships are not accurate and reliable since they depend on assumptions such as distribution selection for each duration; they require a large number of parameters, experience intensive equations and regionalization is also very poor and coarse. In this study, scaling properties of extreme rainfall depth series were examined to establish scaling behavior of statistical moments and quantile estimates over different durations. The annual extreme series of precipitation maxima for storm duration ranging from 0.5 to 24 hr observed at network of rain gauges sited in Oromia regional state were analyzed using an approach based on moments. The analysis investigated the statistical properties of rainfall extremes and detected that the statistics of the rainfall extremes follows a power law relation with its duration. Moreover, the variations of the distribution parameters with durations of annual maximum rainfall depth series were explored and found that the logEV1, EV1 and logistic distribution parameters exhibit a power law relationship with durations. Following the analysis, scale invariance of extreme rainfall depth series is investigated and dissipative (multiple scaling) nature of extreme rainfall depth series is considered, thus introducing a general distribution free framework to develop Depth Duration frequency (DDF) model.

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INTRODUCTION

The scaling or scale-invariant models enable us to transform data from one temporal or spatial model to another one and thus, help us to overcome the difficulty of inadequate data. Therefore, investigation of scaling property of extreme rainfall depth series can provide comprehensive statistical description of the rainfall depth-duration-frequency relationships. A natural process fulfills the simple scaling property if the underlying probability distribution of some physical measurements at one scale is identical to the distribution at another scale multiplied by a factor that is a power function of the ratio of the two scales. A random rainfall depth series with duration D, R(D) exhibits a simple scale invariance behavior if:

$$\lambda D) \overset{d}{=} \lambda^{\eta} I(D) \text{ holds.} \quad (1)$$

The equality $\overset{d}{=}$ refers to identical probability distributions in both sides of the equations; λ denotes a scale factor and η is a scaling exponent. Strict sense simple scaling property asserts that the probability distributions of extreme rainfall depth series at various durations are scale invariance. Further relaxing this preposition, a wide sense simple scaling extends the scale invariant property to quantiles and moments of the extreme rainfall depth series. According to a wide sense simple scaling property, the quantile of extreme rainfall depth series is given by:

$$h_T(\lambda D) = \lambda^{\eta} h_T(D) \quad (2)$$

If we take the 1 hr annual maximum rainfall depth as reference duration, the scale factor (λ=D/1) would be equal to the duration of the extreme rainfall depth at consideration. Therefore, the quantile of the extreme rainfall depth that obeys the wide sense simple scaling property could be given by:

$$h_T(D) = D^{\eta} h_T(1) \quad (3)$$

The distribution function used to explain the 1-hr duration extreme rainfall depth series determines the form of the quantile function in Equation (3). Assuming the scaling exponent is independent of recurrence interval, the row moments and probability weighted moments of the extreme rainfall depth series at order q can be respectively given as

$$E\left[I^{q}(\lambda D)\right] \overset{d}{=} \lambda^{q\eta} E\left[I^{q}(D)\right] \quad (4)$$

$$M_{1,q,0}(\lambda D) = \lambda^{\eta} M_{1,q,0}(D) \quad (5)$$

Where the E[ ] is expected values operator, q is the moment order and λ is a scale factor and M1, q, 0 is the probability weighted moment of order q. The random field I(D) exhibits a simple scale invariants in a wide sense if the above equation holds. As already shown in many literatures , further manipulations on the moments in the above equation (4) and (5) implies that the dimensionless central moments(coefficient of variation, coefficient of skewness, coefficient of kurtosis) and the corresponding dimensionless L-moment ratios of extreme rainfall depth series which exhibits wide sense simple scaling property are independent of duration.

Suppose the wide sense simple scaling holds for duration in the range $D_X \leq D \leq D_Y$ and D_* be reference duration within this range.

The reference duration could be the time scale at which the reference rainfall is measured. If the scale factor is defined as $\lambda = D/D_*$, then the quantiles of extreme rainfall depth series in equation (2) can be expressed as

$$h_T(D) = \frac{h_T(D_*)}{D_*^{\eta}} D^{\eta} \qquad (6)$$

Similar power relationships can be deduced from the row moments and probability weighted moments of extreme rainfall depth series as, respectively

$$E[H_D^q] = \frac{E[H_{D_*}^q]}{D_*^{q\eta}} D^{q\eta} \qquad (7)$$

and

$$\beta_q(D) = \frac{\beta_q(D_*)}{D_*^{\eta}} D^{\eta} \qquad (8)$$

MATERIALS AND METHODS

Description of the Study Area

The study area selected for the study was Oromia regional state which is located between 3^0N to 10.5^0N latitude and 34^0E to 43^0E longitude. It covers an area of 353,690 km^2 and accounts for 32% of the country (population and housing census commission report, 1994). Based on figures from the Central Statistical Agency of Ethiopia (CSA) published in 2005, Oromia has an estimated total population of 26,553,000. The regional state of Oromia borders Afar and Amhara regional states in the north, Kenya in the south, the regional state of Somali in the east, and the Republic of Sudan and the regional state of Benishangul in the west, and the regional states of Southern Nations and Gambela in the south.

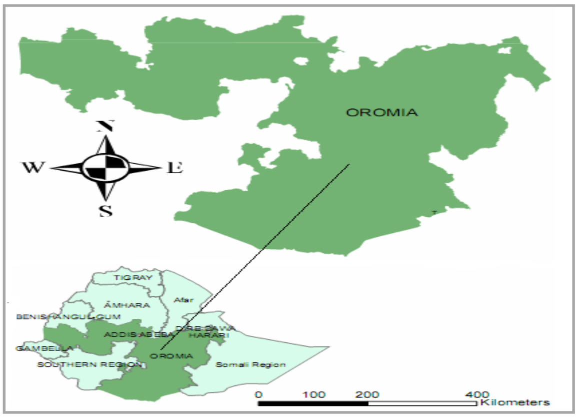

Figure 1: Location of the study area.

Material Used

The materials used for this research were;

- Hourly rainfall data
- Arc GIS to locate the station and delineate the study area
- Spreadsheet/MS Excel 2003 for data analysis and graphical distribution fitting
- Easy fit software for distribution fitting and parameter estimation

Methodology

It is obvious that a random sample of rainfall data is required prior to the implementation of frequency analysis. The random sample often is an annual maximum series of total rainfall depth with respect to a pre-specified duration (D). Before data collection begins, the pre-specified durations, also known as design duration, are artificially designated which are used to determine the corresponding annual maximum rainfall depth in contrast to event durations which are actual raining periods of time of real storm events. The design duration selected for this study was 0.5, 1, 2, 3, 5, 6 and 24 hr. From the data base of 55 stations, annual maximum rainfall depth series from 1978 to 2012 having 32 years length was obtained by employing fixed duration technique at each stations in the study area which involves a continuous moving of a window of size D(duration) along the time axis and selecting the maximum total rainfall values with in the window in each year. After the necessary data were collected, missing rain fall data's were filled and also the collected data series for all stations were checked for consistency, using double mass curve method. The data were also tested for independence and stationarity by a fortran programme based on lag-one serial correlation coefficient test and Wald-Wolfowitz(W-W) respectively. Statistical analysis based on product moments and probability weighted moments was employed to summarize the extreme rainfall data series and to investigate the scaling properties of extreme rainfall depth series in the study region and its efficiency was tested against quantile estimate from logEV1 distribution, observed hourly rainfall depth and empirical IDF estimates previously developed for the region.

RESULT AND DISCUSSION

Statistical Analysis of Extreme Rainfall Depth Series

Rainfall statistics of Oromia Regional State were computed using both conventional moments and L-moment methods. However, L-moment is a powerful and efficient method to compute any statistical parameters, because such methods can give unbiased estimate of sample parameters and also cannot be easily influenced with the presence of outliers (Rao & Hamed, 2000). Generally, the statistical parameters computed include mean, standard deviation, coefficient of variation/L-coefficient of variation, coefficient of skewness/L-coefficient of skewness, and coefficient of kurtosis/L-coefficient of kurtosis.

Conventional Moments

Moments about the origin or about the mean are used to characterize probability distributions. Moments about the origin are the expected values of powers of random variables. For distribution with a probability density function f(x), the rth moment about the origin is given by:

$$\mu'_r = \int_{-\infty}^{\infty} x^r f(x)\,dx, \qquad \mu'_1 = \mu = mean \qquad (9)$$

The central moments μ are computed by:

$$\mu_r = \int_{-\infty}^{\infty} (x-\mu)^r f(x)dx \quad , \quad \mu_1 = 0 \qquad (10)$$

Sample moments m'_r and m_r are given by:

$$m'_r = \frac{1}{N}\sum_{i=1}^{N} X_i^{\,r} \quad , \quad m'_1 = \text{sample mean} \qquad (11)$$

$$m_r = \frac{1}{N}\sum_{i=1}^{N}(X_i - \overline{X})^r \quad , \quad m_1 = 0 \qquad (12)$$

These sample moments are often biased and may be corrected (cunnane, 1989)

$$m_2 = \frac{1}{N-1}\sum (X_i - \overline{X})^2 \qquad (13)$$

$$m_3 = \frac{N}{(N-1)(N-2)}\sum (X_i - \overline{X})^3 \qquad (14)$$

$$m_4 = \frac{N^2}{(N-1)(N-2)(N-3)}\sum (X_i - \overline{X})^4 \qquad (15)$$

The conventional moment ratios are defined as:

$$C_V = \mu_2^{1/2} \big/ \mu_1' \qquad (16)$$

$$C_s = \mu_3 \big/ \mu_2^{3/2} \qquad (17)$$

$$C_k = \mu_4 \big/ \mu_2^{2} \qquad (18)$$

Where: C_V – Coefficient of variation; C_s - coefficient of skewness and C_k – Coefficient of kurtosis

L-moments

L-moments are analogous to conventional moments but are estimated by linear combination of an ordered data set, namely L-statistics (Rao & Hamed, 2000). Like ordinary product moments, L-moment summarizes the characteristics or shape of theoretical probability distributions and observed samples. Both moment types offer measures of distributional location (mean), scale (variance), skewness (symmetric shape), and kurtosis (flatness). L-moment offer significant advantages over ordinary product moments because of the following reasons:

- L-moment ratio estimators of location, scale, and shape are nearly unbiased, regardless of the probability distribution from which the observations arise (Hosking 1990);
- L-moment ratio estimators of L-C_v and L-skew do not have bound which depend on a sample size as do the ordinary product moment ratio estimators of C_V and C-skew (Kirby, 1974);
- L-moment ratio estimators such as L-C_v, L-skew, and L-Kurtosis can exhibit lower bias than conventional product moment ratio especially for highly skewed samples; and
- L-moment ratio diagrams are particularly good at identifying the distributional properties of highly skewed data, whereas ordinary product moments diagrams are almost useless for this task (Vogel and Fennessey, 1993).

Probability weighted moments (PWMs) introduced by Greenwood et al. (1979) are linear function of L-moments. Accordingly, PWMs are defined as:

$$\beta_r = E\left[X\{F(X)\}^r\right]$$

This can be written as;

$$\beta_r = \int_0^1 X(F)F^r dF$$

Where F=F(X) is the cumulative distribution function(CDF) for X, X(F) is the inverse CDF of X evaluated at the probability F and r=1, 2, 3, …is a non negative integer. Where r=0, β_0 is equal to the mean of the distribution μ=E[X]

For any distribution the rth L-moment λ_r is related to the rth (PWM) (Hosking, 1990) via;

$$\lambda_{r+1} = \sum_{k=0}^{r} B_k(-1)^{r-k}\binom{r}{k}\binom{r+k}{k}$$

Moreover the first four L-moments are related to the PWMs using

$\lambda_1 = \beta_0$
$\lambda_2 = 2\beta_1 - \beta_0$
$\lambda_3 = 6\beta_2 - 6\beta_1 + \beta_0$
$\lambda_4 = 20\beta_3 - 30\beta_2 + 12\beta_1 - \beta_0$

Hosking (1990) defined the L-moment ratio as follows:

$L_{cv} = \tau_2 = \lambda_2 / \lambda_1$

L-skew= $\tau_3 = \lambda_3 / \lambda_2$

L-kurtosis= $\tau_4 = \lambda_4 / \lambda_2$

L-moment ratio diagrams can be used to compare sample estimates of the dimensionless L-moments ratios with their theoretical counterparts. The sample product moments and L-moment ratio statistics were estimated from the annual maximum rainfall depth series at all stations and also from their logarithmic transformations. To compute these parameters, spread-sheet (Excel), was used. These parameters were plotted against their time scale for each station and the result confirmed that the statistics varies with duration at all station for both product moments and L-moments (Figure 2, 3 and 4). Later on, these statistics are very important for the selection of the distribution and investigating the scaling property of maximum rainfall depth series in the study region. More over no specific trend or relation were obtained between the statistics of extreme rainfall depth series and elevation. The random variation of the statistics with elevation and its spatial variation over the study region are observed from the statistics versus elevation plot (Figure 4) and map of L-statistics (Figure 5) respectively.

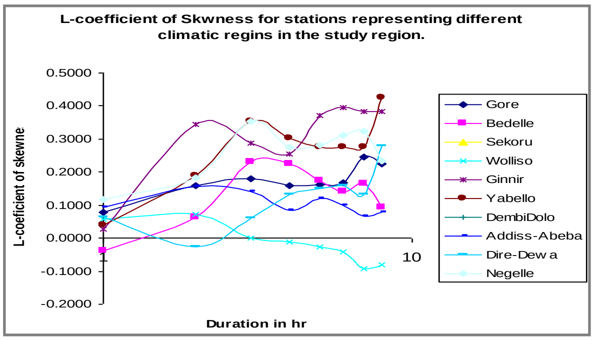

Figure 2: the L-moment ratio (L-coefficient of variation) of maximum rainfall depth series at various durations for sample stations representing different climatic regions in the study area.

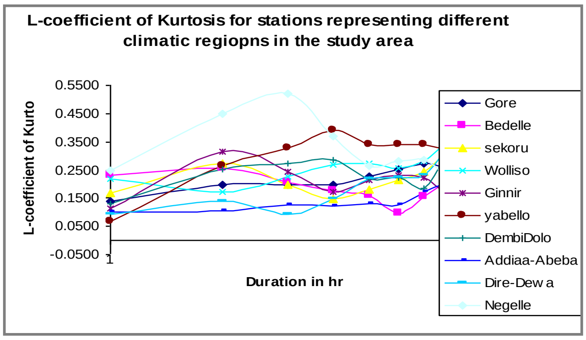

Figure 3: The L-moment ratio (L-coefficient of skewness) of maximum rainfall depth series at various durations for sample stations representing different climatic regions in the study area

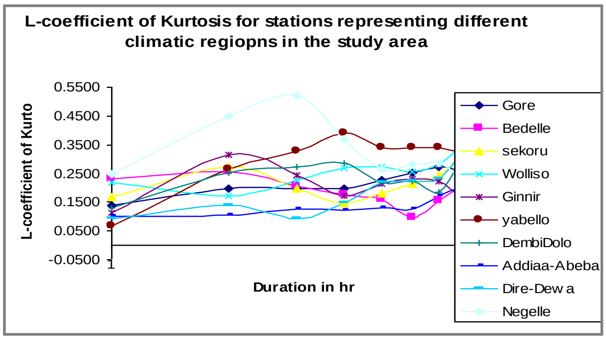

Figure 4: The L-moment ratio (L-coefficient of Kurtosis) of maximum rainfall depth series at various durations for sample stations representing different climatic regions in the study area.

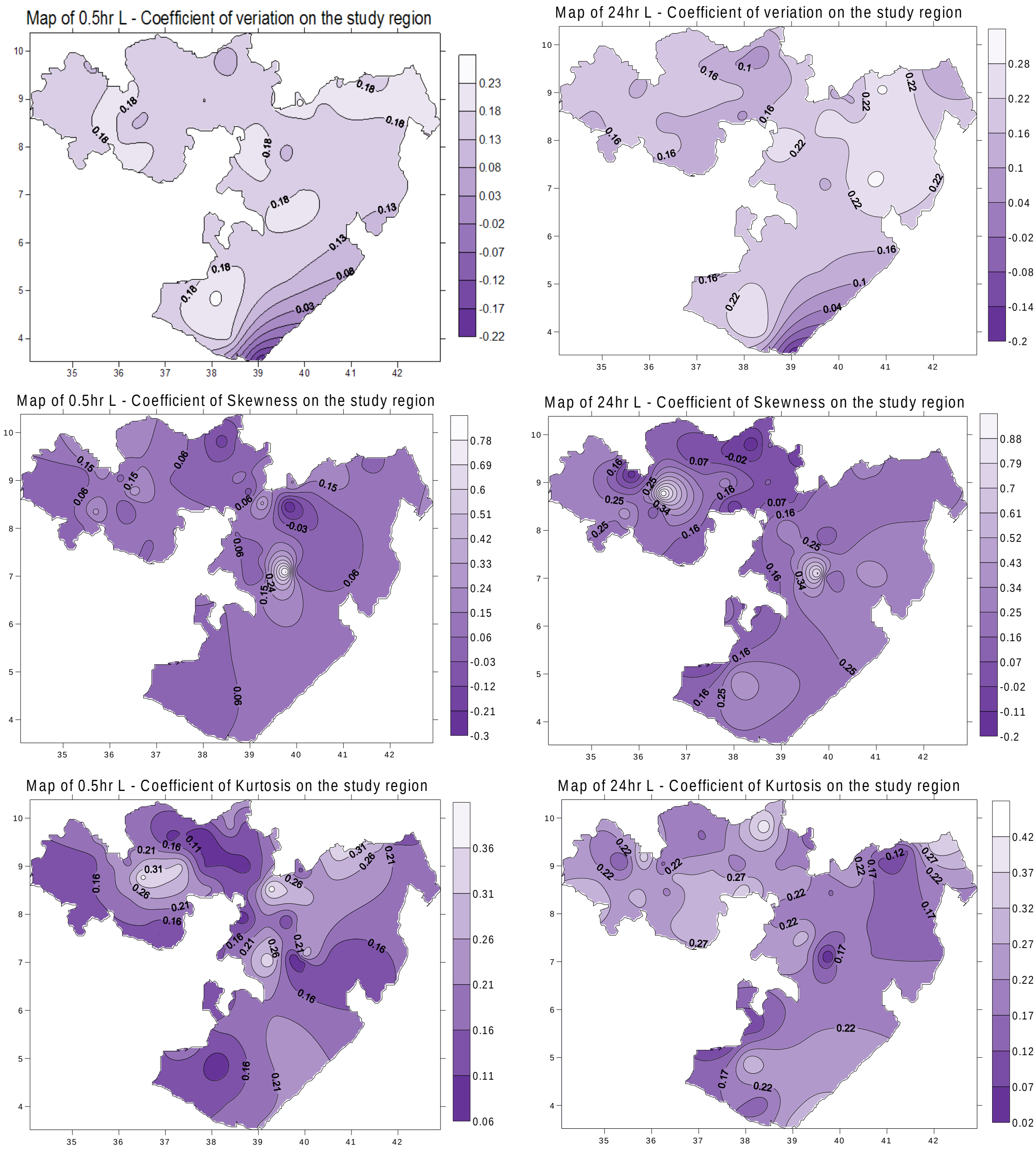

Figure 5: Maps of 0.5hr and 24hr L-statistics in the study area

Investigating the Scaling Property of Extreme Rainfall Depth Series

The scaling property of rainfall in the study region was investigated by computing the moments of extreme rainfall depth series and then by examining the log-log plots of the moments against their duration. The analysis was performed on annual maximum rainfall depth series for storm duration from 0.5hr to 24hr rainfall at all stations. The logarithmic plots of the row moments and the probability weighted moments versus duration for different moment orders at all stations confirmed the existence of power law scaling. For illustration purpose, both the product (conventional) moments and linear moments of extreme rainfall depth series is plotted against their duration in (Figure 6)for Addis Ababa station. This confirms the existence of power law scaling which indicates scaling is applicable in the study region. Therefore, the scaling exponent can be estimated from either the row moments or from the probability weighted moments of the extreme rainfall depth series at various durations in the study region.

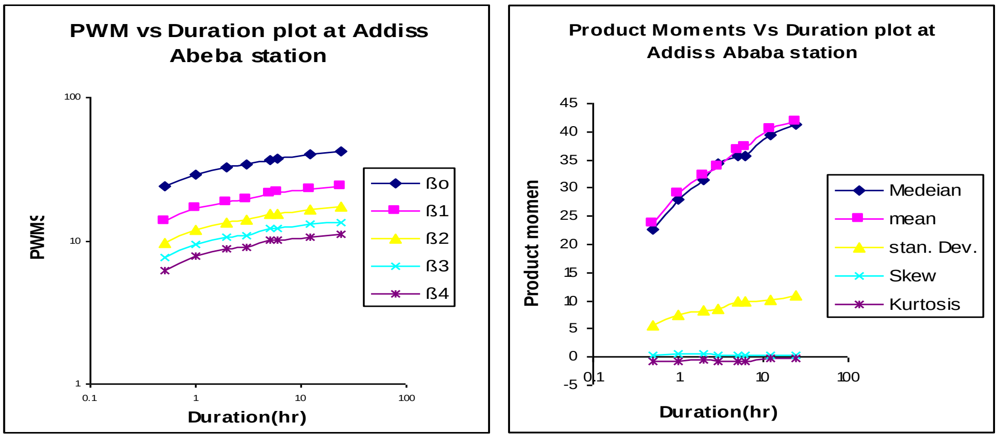

Figure 6: Plot of PWM moments (a) and product moments (b) at Addis Ababa station

Moreover, simple scaling and multi-scaling properties are also deducted in terms of statistical moments. In order to determine if the data follows simple scaling or multi-scaling, the scaling exponent was plotted versus the moment order. As illustrated in Figure 7, the scaling exponent decreases with the moment order at all stations in the study area; and there exists a non-linear relationship between scaling exponent and order of moments. This observation is against the hypothesis of simple scaling theory, and therefore it implies that the property of wide sense simple scaling property of extreme rainfall depth series does not obeyed in the study region.

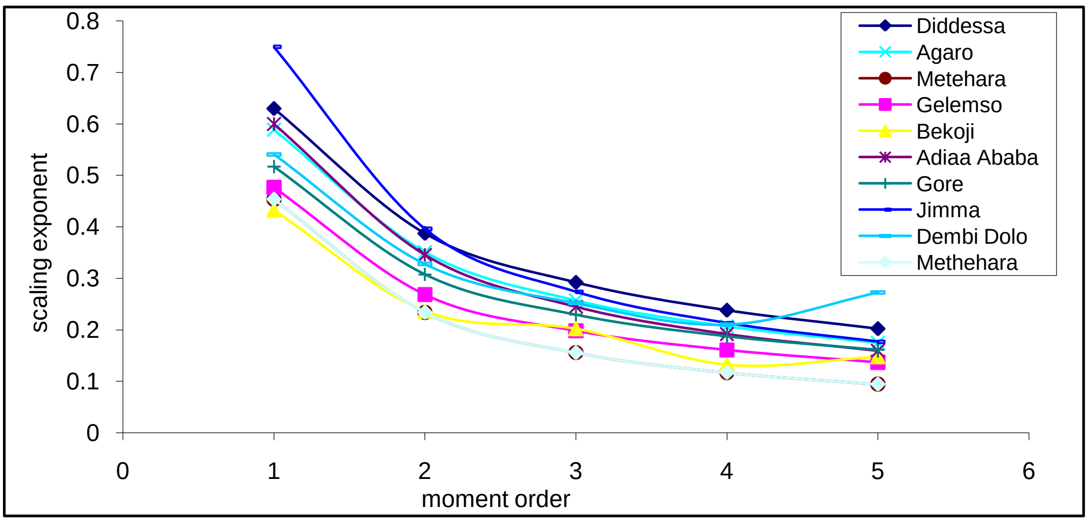

Figure 7: Plot of the scaling exponent versus moment orders of extreme rainfall depth series for sample stations representing different climatic regions

Moreover, the dimensionless central moments (coefficient of variation, skewness, and kurtosis) and the corresponding L-moment ratios of extreme rainfall depth series were drawn and plotted against their duration for all stations. The result shows that dimensionless moment ratios vary with duration for all stations which is against the deduction of wide sense simple scaling theory. Therefore, statistical investigation revealed that the rainfall exhibits multi-scaling property at all stations. The variation of scaling exponent with moment order (Figure 7) may arise from the dependence of scaling exponent on the recurrence interval or duration. In order to investigate the interactions between scaling exponent and recurrence interval, the scaling exponent of annual rainfall depth at all ranks of ascending order at each station were estimated from the scaling quantile relationship of extreme rainfall depth series. The EV1 reduced variate, which is a function of recurrence interval, corresponding to the scaling exponent of each rank of annual rainfall depth was determined from plotting position formula of the ranked extreme rainfall depth series at each station. The variation of scaling exponent with reduced variate is against the deductions of the simple scaling theory that reveals the deviation of the property of the extreme rainfall depth series in the study region from simple scaling behavior as shown in Figure 8.

Moreover, as already explained the dimensionless statistics of extreme rainfall depth series are significantly varies with duration. The variation of scaling exponent with moment order (Figure 7) and recurrence interval (Figure 8) revealed that the annual maximum rainfall depth series are rather exhibiting a wide sense multiple scaling behavior. As already stated in many literatures the quantile function for extreme rainfall depth series exhibiting multiple scaling is given by;

$$h_T(D) = \frac{h_T(D_*)}{D_*^{\delta+\gamma y_T}} D^{\delta+\gamma y_T} \qquad (19)$$

Where D_* is the scale at which reference rainfall is measured.

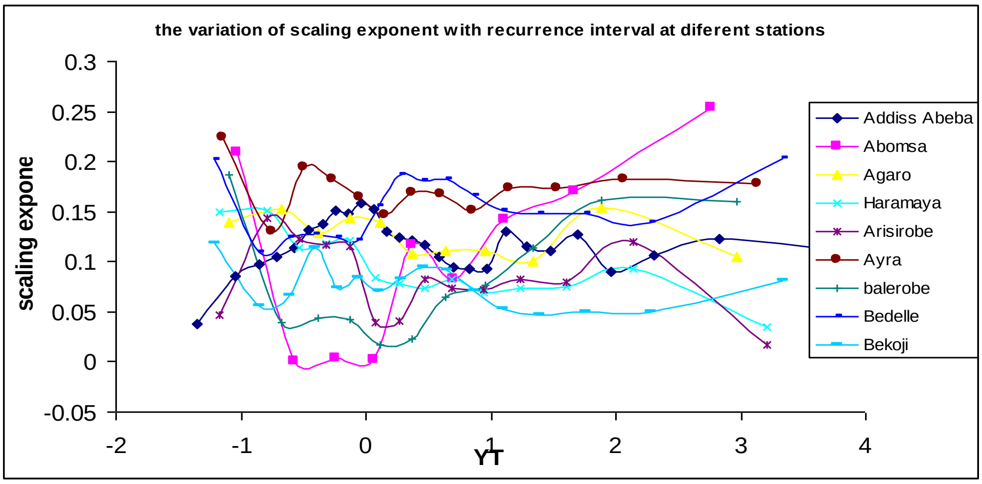

Figure 8: Variation of scaling exponent with EV1 reduced variate

The variation of distribution parameter with durations of annual maximum rainfall depth series are explored for three parameter Log-logistic, two parameter log-logistic, logistic, generalized extreme value, logEV1 and EV1 distributions at fifty five stations in the study region. The method of maximum likelihood is applied for parameter estimation at all stations and for all duration. Graphical evaluation shows that no relationship is observed between the parameter of Log-logistic three parameter distributions and its duration at all stations. All the three parameters (shape, scale and location) parameters are not related to duration (Figure 9). Regarding the two parameter log-logistic distribution, its location parameter exhibits strong power law relation with duration but its scale parameter is not related to duration rather it is more or less constant across duration at all stations.

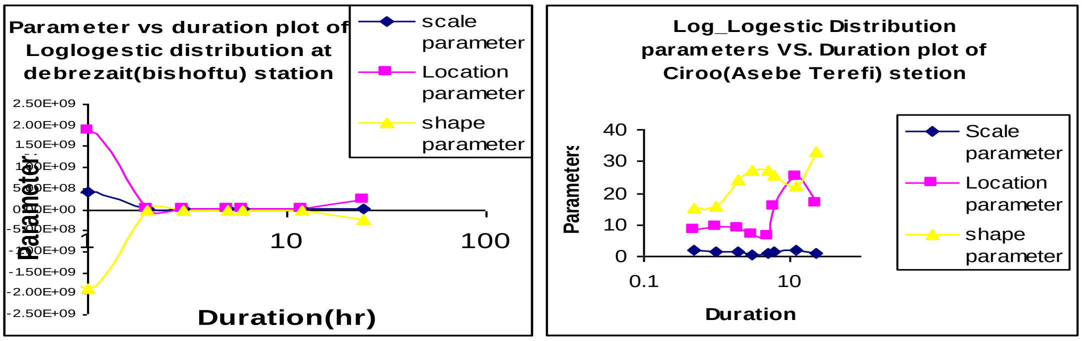

Figure 9: Parameter versus duration plot for three parameter log-logistic distribution

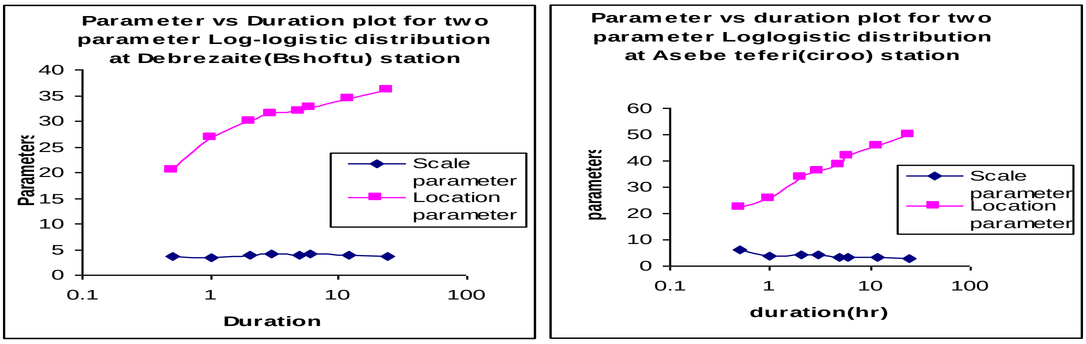

Figure 10: Plot of parameters versus duration for two parameter log-logistic distribution.

In the case of generalized extreme value distribution, location parameter exhibited strong relation with duration and scale parameter weakly related to duration while shape parameter did not exhibit power law relation with duration for all stations in the study region. (Figure 11).

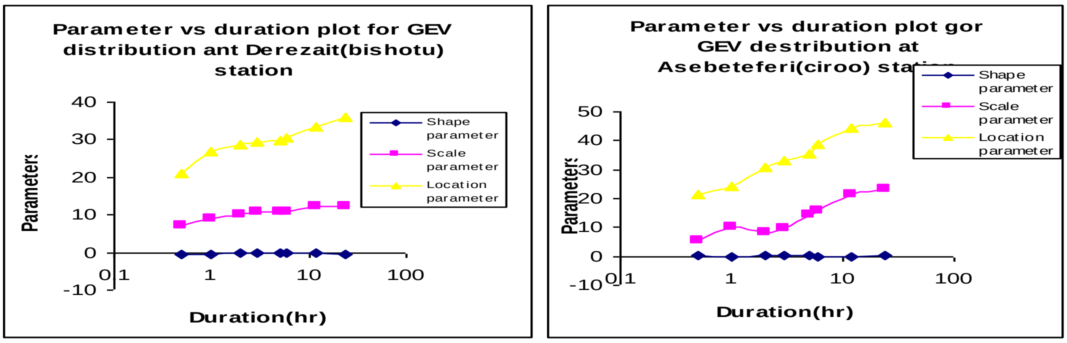

Figure 11: Plot of parameters versus duration for generalized extreme value distribution

The EV1, Logistic and logEV1 distribution parameters exhibited power law relationships with duration for all stations. Sample result for few station is shown in figures 12, 13 and 14 respectively.

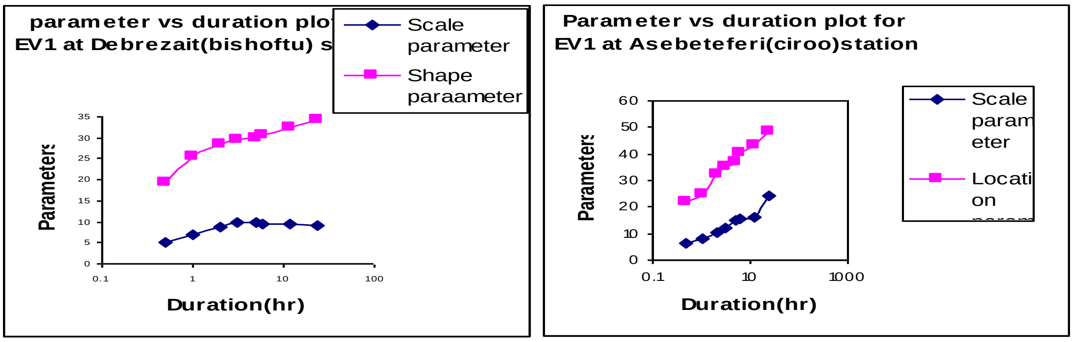

Figure 12: Plot of parameters versus duration for EV1 distribution

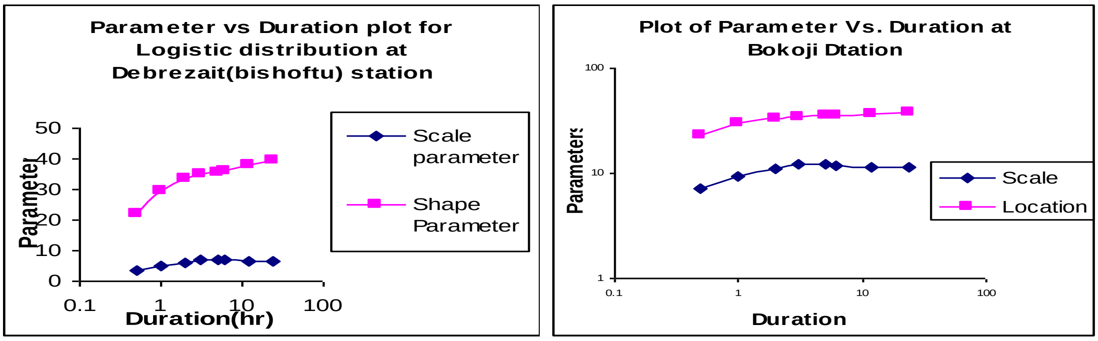

Figure 13: Plot of parameter versus duration for logistic distribution

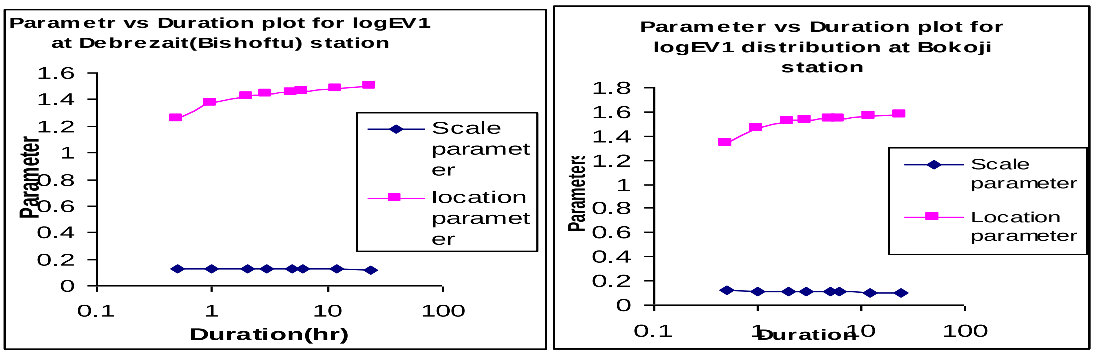

Figure 14: Plot of parameter versus duration for logEV1 distribution

Scaling of the parameters of Logistic, EV1, and logEV1 distribution functions revealed that the parameter-duration scaling properties of extreme rainfall depth series may be explained by these distributions. However the statistical summary of the extreme rainfall depth series shows that the dimensionless coefficient of skewness(conventional skewness coefficient and L-coefficient of skewness) are vary from zero which is against the hypothesis of extreme events following logistic distribution which have a theoretical skewness coefficient of zero. Therefore further investigation will be undertaken on EV1 and logEV1 distributions.

The dependence of EV1 parameters on the duration of the annual maximum rainfall depth series can be expressed as:

$\xi = \psi D^{\phi}$ and $\alpha = \beta D^{\rho}$ (20)

Where ξ and α are respectively location and scale parameters for EV1 distribution. ψ, ϕ, β, ρ are coefficients ;whose coefficients and goodness of fit measures for fifty five stations in the study region are given below.

Table 1: The coefficients and goodness of fit criteria of parameter-duration relationship for EV1 distribution

Stations	Coefficients and efficiency					
	Location, ξ			Scale, α		
	ψ	ϕ	R^2	β	ρ	R^2
Abomsa	26.579	0.118	0.86	6.8805	0.3111	0.9412
Adama	24.003	0.1614	0.96	7.0646	0.1844	0.7656
Addiss Abeba	25.071	0.1409	0.9526	5.5413	0.1579	0.865
Agaro	21.246	0.1657	0.8508	6.1186	0.129	0.4944
Ambo	15.705	0.2131	0.9425	4.1894	0.2471	0.9064
Arisi Adele	19.176	0.1578	0.9648	5.358	0.2081	0.7071
Arisi Robe	26.243	0.104	0.9813	9.4232	0.1315	0.5147
Arjo	27.928	0.2119	0.851	5.5016	0.3708	0.8112
Asossa	25.771	0.1919	0.8741	6.3347	0.068	0.165
Awassa	32.495	0.0848	0.8835	9.4763	0.1413	0.652
Ayira	30.807	0.2029	0.9512	7.7986	0.1986	0.7986
Bale-Robe	22.552	0.1256	0.7919	5.1869	0.2305	0.9276
Bedelle	27.249	0.1533	0.9735	4.6733	0.2078	0.906
Bekoji	28.711	0.1117	0.7974	9.3539	0.1038	0.5427
Ciroo	26.218	0.2114	0.9682	8.2872	0.3293	0.9768
Debre-Brehan	19.077	0.1898	0.9872	5.3943	0.2101	0.9198
Debre-Markos	23.392	0.1403	0.9245	8.467	0.2329	0.7014
Deder	25.705	0.1462	0.911	7.3769	0.2556	0.915
Degehabour	26.533	0.0869	0.9694	8.6101	0.0642	0.5739
Dembi-Dolo	26.208	0.1239	0.8162	4.5672	0.1511	0.6912
Didessa	26.125	0.1372	0.8872	8.5058	0.1562	0.8308
Dilla	31.337	0.1435	0.959	8.3743	0.1953	0.9414
Diredewa	24.044	0.0981	0.8763	5.6389	0.0796	0.4163
Fiche	26.925	0.1579	0.9567	6.0113	0.1291	0.7675
Garba-Guracha	29.052	0.0577	0.9728	3.5241	-0.0298	0.1564
Gelemso	26.039	0.1042	0.8331	9.1497	0.1776	0.7686
Gewane	29.459	0.2108	0.8785	7.9754	0.0761	0.1096
Gimbi	23.425	0.1161	0.8278	6.1998	0.3776	0.9148
Ginnir	19.354	0.1759	0.9446	6.076	0.0607	0.2179
Goba	29.935	0.0945	0.9335	6.8904	0.1214	0.8585
Gore	21.61	0.1837	0.9015	5.1043	0.2151	0.7683
Harere-Mariam	29.655	0.129	0.9488	7.6495	-0.0335	0.3271
Haramaye	22.533	0.1498	0.8088	4.7438	0.1969	0.7855
Hunte	29.132	0.123	0.7848	4.834	0.155	0.6809
Hurso	26.854	0.1758	0.9098	7.4831	0.1486	0.8738
Humuru	22.639	0.1677	0.9299	3.9335	0.2827	0.795
Ijaji	27.797	0.1494	0.9684	5.3341	0.1158	0.7037
Jimma	27.404	0.1426	0.9047	5.0903	0.0194	0.0974
Kachisse	26.061	0.1036	0.9258	5.4836	0.0747	0.8367
Konso	23.367	0.11	0.7337	7.814	0.2083	0.8246
Kulumsa	20.649	0.1437	0.9358	6.8723	0.2672	0.901
Mega	24.056	0.1433	0.8392	7.6233	0.0137	0.137
Methehara	24.585	0.1684	0.9782	7.7455	0.1487	0.6884
Mierab-Abaya	27.209	0.0549	0.8955	9.3078	0.1611	0.7722
Miesso	29.103	0.1669	0.8781	7.0868	0.0659	0.2269
Moyalle	27.064	0.0829	0.7617	5.0343	0.1545	0.7182
Negelle	27.058	0.1931	0.9845	6.613	0.2236	0.7284
Shambu	20.015	0.2553	0.9453	7.2554	0.2478	0.8521
Sinana	28.437	0.1314	0.9294	6.4445	0.0462	0.4517
Sokoru	18.869	0.0965	0.8109	6.2187	0.125	0.7318
Yabello	23.783	0.1421	0.9046	7.378	0.2013	0.9185
Zeway	23.818	0.1246	0.9149	7.4279	0.245	0.9258

More over the semi-log plot in figure 14 revealed that the logEV1 distribution parameters are also related to the durations of extreme rainfall depth series at all stations.

The relationships between extreme rainfall durations and the locations and scale parameters of logEV1 distribution can be expressed respectively as

$\omega = \log(\sigma D^{\delta})$ and $\varphi = \log(\theta D^{\chi})$ (21)

where; ω and φ are location and scale parameters of logEV1 distribution and σ, δ, θ, χ are coefficients whose coefficient and goodness of fit measures at all stations are given below

Table 1: The coefficients and goodness of fit criteria of parameter-duration relationships for logEV1 distribution.

	Coefficients and Efficiency					
	Location, ω			Scale, φ		
Stations	σ	δ	R^2	θ	χ	R^2
Abomsa	1.4314	0.0262	0.8641	0.1086	0.0917	0.6047
Adama	1.374	0.0459	0.9445	0.0853	-0.2809	0.8403
Addiss Abeba	1.3865	0.0417	0.9438	0.0885	0.0272	0.5146
Agaro	1.2989	0.075	0.9569	0.1079	-0.1062	0.5261
Ambo	1.1755	0.0763	0.9146	0.1301	-0.1432	0.4734
Arisi Adele	1.3925	0.053	0.6841	0.1353	-0.0282	0.6177
Arisi Robe	1.4386	0.0588	0.804	0.0707	0.1724	0.7735
Arjo	1.4386	0.0588	0.81	0.0707	0.1724	0.74
Asosa	1.3937	0.0566	0.88	0.0991	0.0899	0.61
Awassa	1.4856	0.0207	0.36	0.1128	0.0652	0.21
Ayira	1.4699	0.0641	0.93	0.1023	0.0726	0.52
Bale-Robe	1.3399	0.0419	0.77	0.0895	0.0237	0.47
Bedelle	1.4335	0.0462	0.87	0.0696	0.0244	0.27
“Bishoftu	1.3497	0.0414	0.81	0.1284	0.0139	0.87
Bekoji	1.4396	0.0359	0.77	0.1166	-0.05	0.73
Ciroo	1.4052	0.0576	0.94	0.1111	0.1175	0.83
Deder	1.4179	0.0416	0.76	0.101	0.0564	0.52
Dembi-Dolo	1.4033	0.0396	0.79	0.0765	0.0259	0.54
Fiche	1.5358	0.0245	0.84	0.0848	0.0715	0.51
Garba-Guracha	1.4567	0.0171	0.96	0.0636	-0.1505	0.76
Gelemso	1.3978	0.032	0.96	0.0936	-0.0622	0.67
Gimbi	1.4478	0.0602	0.88	0.1106	-0.098	0.48
Ginnir	1.3604	0.0338	0.7761	0.0936	0.1869	0.9712
Goba	1.2609	0.0617	0.927	0.1296	-0.1663	0.605
Gore	1.47	0.0311	0.7384	0.0926	-0.1251	0.755
Hagere-Mariam	1.318	0.058	0.0884	0.0941	0.0146	0.0327
Haramaye	1.4614	0.0394	0.9409	0.09931	0.1682	0.9658
Hunte	1.3353	0.0501	0.7817	0.0881	0.0188	0.5503
Humuru	1.4191	0.0521	0.8922	0.0997	-0.097	0.5773
Ijaji	1.3476	0.0521	0.9138	0.0682	0.0615	0.6461
Jimma	1.4339	0.0441	0.9634	0.0829	-0.0854	0.8175
Kachisse	1.4269	0.042	0.8875	0.0766	-0.0916	0.6731
Kulumsa	1.335	0.0419	0.6413	0.1299	-0.0147	0.417
Mega	1.2786	0.0468	0.9344	0.1416	0.0558	0.5219
Methehara	1.3666	0.046	0.8087	0.1083	-0.1072	0.7649
Mirab-Abaya	1.3716	0.0508	0.9801	0.1408	-0.1153	0.7257
Miesso	1.421	0.015	0.8681	0.1157	0.0824	0.5883
Moyalle	1.4499	0.0501	0.8724	0.0957	-0.0938	0.5258
Negelle	1.4284	0.0248	0.7289	0.0685	0.0554	0.2583
Nedjo	1.4035	0.0594	0.9386	0.0751	1366	0.719
Shambu	1.4263	0.0541	0.9751	0.0853	0.0463	0.2127
Sinana	1.2666	0.0863	0.9073	0.1396	-0.0662	0.5946
Sokoru	1.445	0.0381	0.9076	0.0841	-0.0547	0.6691
Wolliso	1.3893	0.0656	0.7955	0.0678	-0.2076	0.5866
Yabello	1.2585	0.0364	0.7614	0.1177	-0.0278	0.1072
Zeway	1.3542	0.0343	0.8808	0.1198	0.11	0.9376

Moreover, the quantile function of the annual maximum rainfall depth series can be expressed in terms of its duration and recurrence interval by substituting the parameter-duration relationships into the inverse of cumulative distribution function. Recall that for EV1 distribution the inverse distribution function (quantile function) is given by:

$$h_T(D) = \xi + \alpha y_T \quad (22)$$

where y_T is EV1 reduced vitiate and ξ and α are location and scale parameters respectively. Accordingly, direct substitution of parameter duration relationship (20) in to EV1 quantile function (22) gives complete description of the EV1 quantile function give by;

$$h_T(D) = \psi D^{\phi} + \beta D^{\rho} y_T \quad (23)$$

Similarly, after substitutions and algebraic manipulations the complete description of the quantile functions of the logEV1 distribution can be expressed as:

$$h_T = \sigma\theta^{yT} D^{\delta+\tau y_T} \quad (24)$$

The quantile functions (23 and 24) are often referred to as the rainfall depth duration frequency relationships. The power law dependence of the distribution parameters and quantiles on the durations of extreme rainfall depth series shows that the extreme rainfall depth series exhibit scaling property with respect to the duration of occurrence. Recall also that the statistical investigation of extreme rainfall depth series confirms the multi scaling property of the rainfall depth series in the study region. While the rainfall depth duration frequency relationships derived from multiple scaling behavior (equation 19) is very similar to the quantile function of the logEV1 distribution (equation 24), that derived from simple scaling property (6) has identical features with the quantile function of the EV1 distribution (equation 23). Therefore, the logEV1 distribution is the most appropriate frequency distribution for extreme rainfall depth series that exhibits multiple scaling properties.

CONCLUSIONS

One of the objectives of this study was to investigate the scaling properties of extreme rainfall depth series in the study region. The collected extreme rainfall depth series data were checked for consistency using mass curve analysis method; Test for independence were made by Fortran program based on (W-W) test; and the independency and consistency of the extreme rainfall depth series were confirmed. The study summarized the statistical properties of extreme rainfall depth series using an approach based on ordinary and L-moments, and detected important relationship between these statistics and durations of extreme rainfall depth series. The investigation revealed that the statistics of rainfall extremes vary systematically with duration. This variation of the statistics of rainfall extremes were used to investigate the scaling properties of the extreme rainfall depth series in the study region. The variation of distribution parameters with durations of annual maximum rainfall depth series were explored for different distributions. From the result it was confirmed that the EV1, logEV1 and logistic distribution parameters exhibited power law relationships with duration. Similar relationships were also obtained for location and scale parameters of GEV distribution while the shape parameter of GEV distribution was not related to the duration of extreme rainfall depth series. For the rest of the distribution functions tested in the region, power law relation did not exist. Furthermore from the logarithmic plots of ordinary moments and probability weighted moments versus duration for different moment orders at all stations, it was confirmed that power law scaling has existed.

From this, it was found out that the property of the time scale invariance of extreme rainfall quantiles in the study region follow a multiple scaling property. It was also found out that the investigated property of the extreme rainfall depth series has significant practical importance, because statistical extreme rainfall inference can be made to further develop a Depth Duration Frequency model that robustly explains the rainfall depth duration frequency relationship in the study region.

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