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Original Research

Statistical and Squeezing Properties of Superposed Squeezed Vacuum States

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Abstract

This research looks at the compressed light beams superposed on top of each other and their statistical and compressive properties. It is possible to construct the anti-normally ordered characteristics function using the density operator of single-mode squeezed vacuum states. The quasi-probability distribution functions (Q-function) for identical two-mode superposed states, three-mode squeezed vacuum states, and one-mode squeezed vacuum states can be obtained using this function. We calculate the statistical and squeezing characteristics of single mode and superposed light beams with the function Q that was derived. The correlation between the average photon number and quadrature variance of the superimposed light beams and their corresponding values for the average photon number and quadrature variance of the single-mode compressed vacuum states is found to be equal to zero. The average amount of photons also rises as the squeezing parameter is increased. In addition, we find that the superposed light beams' quadrature squeezing is the same as the one in the single-mode squeezed vacuum. As the squeeze parameter gets closer to infinity, the plus quadrature, where the compression occurs, approaches unity.

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INTRODUCTION

Chaos, coherence, and compression are the three main quantum states of light. The super-Poissonian photon statics chaotic state of light is one of its classical features. It includes thermal light, for example. The coherence state is a specific superposition of photon-free number states determined by the lowest uncertainty and poissonian photon statistics. Lumbropoulos and Petrosyan (2007), Lü (1999), and Barnett (2002) all note that the compressed state is a well-known and studied non-classical property of light.

Quadrature operators, formed by constructing and destroying Hamiltonian arrangements of operators, reflect the observed values of a single-mode light. When the noise in one quadrature is smaller than the coherence state level while the noise in the conjugate quadrature increases, and the uncertainty relation is preserved, the single-mode light is said to be in a squeezed state (Scully & Zubairy, 1997). Whenever the quadrature uncertainties satisfy the minimal uncertainty relation, we say that the light mode is in a minimal uncertainty state. To be in a minimum uncertainty state, an SVS must have quadrature variances that are fewer than the level of coherence and conjugate variances that are more than the level of coherent.

Numerous optical processes, including subharmonic production and second harmonic generation, result in squeezed states (Darge & Kassahun, 2010; Gardiner & Zoller, 2004). Squeezed light with reduced noise might be advantageous for optical communications and precise measurement. Using the standard concept of quadrature variance, these authors carried out their research (Kassahun, 2012). In contrast, the quadrature variance makes sense for superposed light beams. His findings indicate that the quadrature variances of the coherent and compressed lights add up to the quadrature variance of the superposed lights (Yamamoto & Haus, 1986; Yuen & Shapiro, 1978).

It is common practice to use distribution functions to characterise the quantum characteristics of light modes. They are therefore perfectly appropriate for assessing the expected values of the operators' symmetrically ordered, anti-symmetrically ordered, and normally ordered functions (Chuma, 2016; Fan, 2011).

The Q distribution function is used in this work, and the statistical and squeezing characteristics of the superposed three identical SMSVSs are discussed.

The Condition of Squeezed Vacuum (SVS)

Vacuum state with a single mode of compression Essentially, a degenerate parametric amplifier is a single-mode squeezed vacuum state that is powered by coherent light and consists of nonlinear crystals. The classical treatment of a strong pump mode can be approximated to a good degree; this is the shape of the degenerate parametric amplifier's Hamiltonian.

$$\hat{H} = \frac{i\varepsilon}{2}(\hat{a}^2 - \hat{a}^{\dagger 2}) \quad (1)$$

Initially in vacuum state $|0\rangle$, the state vector of the single mode can be represented as,

$$|\psi(t)\rangle = \exp[\frac{i\varepsilon}{2}(\hat{a}^2 - \varepsilon\hat{a}^{\dagger 2})]\,|0\rangle \quad (2)$$

and the SVS $|r\rangle$ is defined as

$$|r\rangle = \hat{S}^{\dagger}(r)|0\rangle, \quad (3)$$

Where

$$\hat{S}^{\dagger}(r) = exp\left[\frac{ir}{2}(\hat{a}^2 - \hat{a}^{\dagger 2})\right] \quad (4)$$

is the squeeze operator, with $\varepsilon = \lambda\beta_0$ (λ is the coupling constant and β_0 is assumed to be real and positive constant) and r being the squeeze parameter taken to be real and a positive constant. The corresponding evolution operator of the single-mode read

$$\hat{a}(r) = \hat{S}^{\dagger}(r)\hat{a}\hat{S}(r) = \hat{a}^{\dagger}\cosh r - \hat{a}\sinh r \quad (5)$$

and its complex conjugate takes the form

$$\hat{a}^{\dagger}(r) = \hat{S}(r)\hat{a}\hat{S}^{\dagger}(r) = \hat{a}\cosh r - \hat{a}^{\dagger}\sinh r. \quad (6)$$

The anti-normally ordered characteristic function

It is defined as

$$\phi_a(z) = Tr\left(\hat{\rho}\, e^{-z^*\hat{a}} e^{z\hat{a}^{\dagger}}\right) \quad (7)$$

Here, $(\hat{\rho} = |r\rangle\langle r|)$,

$$\phi_a(z) = Tr\left(|r\rangle\langle r|\, e^{-z^*\hat{a}} e^{z\hat{a}^{\dagger}}\right) \quad (8)$$

Use of Eq. (2) along with the unitary operator has been made. Using a power series expansion, Eq. (7) ,

$$\phi_a(z) = \langle 0|\hat{S}^{\dagger}(r)e^{-z^*\hat{a}}\hat{S}(r)\hat{S}^{\dagger}(r)e^{z\hat{a}^{\dagger}}\hat{S}(r)|0\rangle \quad (9)$$

Employing the Baker-Hausdorff identity

$$e^{\hat{A}}e^{\hat{B}} = exp\left(\hat{A} + \hat{B} + \frac{1}{2}[\hat{A}, \hat{B}]\right) \quad (10)$$

$$\phi_a(z) = e^{\frac{-z^*z}{2}}\langle 0|e^{z\hat{a}^\dagger(r) - z^*\hat{a}(r)}|0\rangle. \quad (11)$$

Use of Eqs. (4) and (5), the result obtained was

$$\phi_a(z) = e^{\frac{-z^*z}{2} - \frac{FG}{2}}\langle 0|e^{F\hat{a}^\dagger - G\hat{a}}|0\rangle, \quad (12)$$

Where F= zcosh r +z*sinh r and G=z*cosh r +z sinh r. With the help of the identity

$$e^{\hat{A}}e^{\hat{B}} = e^{\hat{A}}e^{\hat{B}}e^{[\hat{A},\hat{B}]} \quad (13)$$

in Eq. (12), we found

$$\phi_a(z) = \exp\left[-z^*z\, cosh^2 r - \frac{1}{2} sinhr \cosh r\,(z^2 + z^{*2})\right] \quad (14)$$

The Q Function

It is defined by

$$Q(\alpha^*, \alpha) = \frac{1}{\pi}\langle\alpha|\hat{\rho}|\alpha\rangle \quad (15)$$

$$\phi_a(z) = \int d^2\alpha\, Q(\alpha^*, \alpha) exp(z\alpha^* - z^*\alpha) \quad (16)$$

The Q function is the inverse Fourier transform of Eq. (16) and is given by

$$Q(\alpha^*, \alpha) = \frac{1}{\pi^2}\int d^2\alpha\, \phi_a(z)\, exp(z^*\alpha - z\alpha^*). \quad (17)$$

Now on combining Eq. (13) with Eq. (16), there follows

$$Q(\alpha^*, \alpha) = \frac{1}{\pi^2}\int d^2\alpha\, exp[-az^*z + bz + cz^* + A(z^2 + z^{*2})] \quad (18)$$

where a=cosh^2 r, b = -α^*, $c = \alpha$, and A= $-\frac{1}{2}\sinh r \cosh r$. Employing the standard integral relation

$$\frac{1}{\pi}\int d^2\alpha\, exp[-az^*z + bz + cz^* + Az^2 + Bz^{*2}] = \frac{1}{\sqrt{a^2 - 4AB}} exp\left(\frac{abc + Ac^2 + Bb^2}{a^2 - 4AB}\right), a > 0 \quad (19)$$

Using Eq. (18), we found

$$Q(\alpha^*, \alpha) = \frac{sechr}{\pi}\, exp\left[-\alpha^*\alpha - \frac{1}{2}\tanh r\,(\alpha^2 + \alpha^{*2})\right] \quad (20)$$

is the normalized Q function for an SMSVS.

Photon Statistics

The density operator allows one to express the anticipated value of operator function Dˆas

$$\hat{D} = Tr\left(\hat{\rho}\,\hat{D}\right) \quad (21)$$

$$\langle\hat{D}\rangle = \int d^2\alpha\, Q(\alpha^*, \alpha) D_a(\alpha, \alpha^*), \quad (22)$$

where $Q(\alpha^*, \alpha)$ is given in Eq. (12) and $D_a(\alpha, \alpha^*)$ is the operator function $D_a(\hat{a}, \hat{a}^\dagger)$ in the anti-normal order corresponding to the c-number function. In view of Eq. (20), Eq. (22) can be rewritten as

$$\langle\hat{D}\rangle = \frac{sechr}{\pi}\int d^2\alpha\, exp\left[-\alpha^*\alpha - \frac{1}{2}\tanh r\,(\alpha^2 + \alpha^{*2})\right] D_a(\alpha, \alpha^*). \quad (23)$$

One way to express the average photon number is

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{sechr}{\pi}\int d^2\alpha\, exp\left[-\alpha^*\alpha - \frac{1}{2}\tanh r\,(\alpha^2+\alpha^{*2})\right](\alpha\alpha^* - 1), \quad (24)$$

$$\langle\hat{a}^\dagger\hat{a}\rangle = sinh^2 r. \quad (25)$$

Furthermore, the photon number variance for an SMSVS is

$$(\Delta n)^2 = 2sinh^2 r\,(sinh^2 r + 1). \quad (26)$$

Quadrature Squeezing

The variance of the quadrature operators for an SMSVS is defined by

$$(\Delta \mathrm{d}\pm)^2 = 1 + \langle:\hat{d}\pm, \hat{d}\pm:\rangle, \quad (27)$$

Using the two Hermitian quadrature operators,

$$\hat{d}+ = \hat{a} + \hat{a}^\dagger \quad (28)$$

and

$$\hat{d}- = i(\hat{a}^\dagger - \hat{a}), \quad (29)$$

Eq. (27) can be rewritten as

$$(\Delta \mathrm{d}\pm)^2 = 1 + 2\langle\hat{a}^\dagger\hat{a}\rangle \pm \langle\hat{a}^2\rangle \pm \langle\hat{a}^{\dagger 2}\rangle \mp \langle\hat{a}^\dagger\rangle^2 \mp \langle\hat{a}\rangle^2 - 2\langle\hat{a}^\dagger\rangle\langle\hat{a}\rangle \quad (30)$$

Using Eq. (23), the computed expectation values are

$$\langle\hat{a}^\dagger\rangle = \langle\hat{a}\rangle = 0 \quad (31)$$

$$\langle\hat{a}^2\rangle = \langle\hat{a}^{\dagger 2}\rangle = -\frac{1}{4}(e^{2r} - e^{-2r}). \quad (32)$$

Substituting Eqs. (31) and (32) into Eq. (30), we obtained

$$(\Delta \mathrm{d}\pm)^2 = e^{\mp 2r}. \quad (33)$$

Thus, for big positive values of r, the plus quadrature operator's variance is less than the coherence level, and the negative quadrature fluctuation is accentuated. The result is that the plus quadrature is the location of the squeezed state of SMSVS. Using Eq. (30), it is clear that the quadrature variance of a coherent state can be expressed as unit. Next, an SMSVS's quadrature squeezing as

$$s = \frac{1-(\Delta \mathrm{d}+)^2}{1}. \quad (34)$$

In view of Eq. (33), we see that

$$s = 1 - e^{-2r}. \quad (35)$$

The quadrature squeezing vanishes at zero value of the squeeze parameter r. As the squeeze parameter approaches infinity, the quadrature squeezing approaches one. That is the squeezing becomes 100%.

The Superposed Squeezed Vacuum States

The Q function for two SMSVS

Suppose $Q_1(\alpha^*, \alpha) = Q(\beta^*, \alpha\text{-}\gamma)$ and $Q_2(\alpha^*, \alpha) = Q(\gamma^*, \alpha - \beta)$ are two light beams expressed as

$$Q(\beta^*, \alpha-\gamma) = \frac{sechr}{\pi}\, exp\left[-\beta^*(\alpha-\gamma) - \frac{1}{2}\tanh r\,(\beta^{*2} + \alpha^{*2} + \gamma^2 - 2\alpha\gamma)\right] \quad (36)$$

and

$$Q(\gamma^*, \quad \alpha-\beta) = \frac{sechr}{\pi}\, exp\left[-\gamma^*(\alpha-\beta) - \frac{1}{2}\tanh r\,(\beta^2 + \alpha^2 + \gamma^{*2} - 2\alpha\gamma)\right]. \quad (37)$$

Now we need to determine Q (α^*, α) function for the superposed two light beams that can be written as

$$Q_{12}(\alpha^*,\alpha) = \frac{1}{\pi}\iint d^2\beta d^2\gamma\, Q\,(\beta^*,\alpha-\gamma)Q\,(\gamma^*,\alpha-\beta) \times exp[\alpha\beta^* + \beta\alpha^* + \gamma(\alpha^*-\beta^*) + \gamma^*(\alpha-\beta) - \alpha^*\alpha - \beta^*\beta - \gamma^*\gamma]. \quad (38)$$

Substituting Eqs. (35) and (36) into Eq. (37), using Eq. (18) we arrive at

$$Q_{12}(\alpha^*,\alpha) = \frac{1}{\pi} exp[-\alpha^*\alpha(1+2sinh^2r) - sinhr\cosh r\,(\alpha^2+\alpha^{*2})]. \quad (39)$$

The Q function for three identical SMSVS

By the account of Eqs. (19) and (38), consider

$$Q_{12}(\beta^*,\alpha-\gamma = \frac{1}{\pi}\; exp[-\beta^*(\alpha-\gamma)(1+2sinh^2r) - sinhr\cosh r\;\,(\beta^{*2}+\alpha^{*2}+\gamma^2-2\alpha\gamma)] \quad (40)$$

and

$$Q_3(\gamma*,\alpha-\beta) = \frac{sechr}{\pi}\; exp\left[-\gamma^*(\alpha-\beta) - \frac{1}{2}\tanh r\,(\beta^2+\alpha^2+\gamma^{*2}-2\alpha\beta)\right] \quad (41)$$

$$Q(\alpha^*,\alpha) = \frac{1}{\pi}\iint d^2\beta d^2\gamma\, Q_{12}(\beta^*,\alpha-\gamma)Q_3(\gamma*,\alpha-\beta) \times exp[\alpha\beta^*+\beta\alpha^*+\gamma(\alpha^*-\beta^*)+\gamma^*(\alpha-\beta)-\alpha^*\alpha-\beta^*\beta-\gamma^*\gamma]. \quad (42)$$

After the substituting Eqs. (39) and (40) into Eq. (41), employing Eq. (18) and integrating the result becomes

$$Q(\alpha^*,\alpha) = \frac{1}{\pi\sqrt{1-3sinh^2r}} exp[-a\alpha^*\alpha + A(\alpha^2+\alpha^{*2})], \quad (43)$$

where

$$a = \frac{1+3sinh^2r}{1-sinh^2r} \quad (44)$$

$$A = -\frac{3}{2}\left(\frac{\sinh r\cosh r}{1-3sinh^2r}\right) \quad (45)$$

This is a normalized Q function for the superposed three identical SMSVSs.

Photon statistics of the superposed three identical SMSVS

As a result, we can find the average photon number and the variation in photon number using the superposed Q function (Eq. 43). In addition, we analyze the squeezing of its quadrature. The average photon number for the superposed states can be determined by substituting Eq. (43) into Eq. (22), yielding

$$\langle\hat{D}\rangle = \frac{1}{\pi\sqrt{1-3sinh^2r}} \int d^2\alpha\, exp[-a\alpha^*\alpha + A(\alpha^2+\alpha^{*2})]D_a(\alpha,\alpha^*) \quad (46)$$

where a and A are given by Eqs. (44) and (45). $D_a(\alpha,\alpha^*)$, is the c-number function corresponding to operator $\hat{D}$ in the anti-normal order. The mean number $\langle\hat{a}^\dagger\hat{a}\rangle$ can be expressed as

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{1}{\pi\sqrt{1-3sinh^2r}}\; \int d^2\alpha\, exp\big[exp[-a\alpha^*\alpha + A(\alpha^2+\alpha^{*2})]\big](\alpha\alpha^*-1). \quad (47)$$

This can be rewritten as

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{1}{\pi\sqrt{1-3sinh^2r}}\; \int d^2\alpha\, exp\big[exp[-a\alpha^*\alpha + b\alpha + c\alpha^* + A(\alpha^2+\alpha^{*2})]\big](\alpha\alpha^*-1)|b=c=0. \quad (48)$$

And after completing the integration, we go to

$$\langle\hat{a}^{\dagger}\hat{a}\rangle = \frac{a^2-4A^2}{\sqrt{1-3sinh^2r}} \left(\frac{\partial^2}{\partial b\partial c} - 1\right) exp\left[\frac{abc+A(c^2+b^2)}{a^2-4A^2}\right] |b = c = 0. \quad (49)$$

So that completing the separation, applying the condition b=c=o, and substituting the values of a and A, one effectively gets (Hach, 1993).

$$\langle\hat{a}^{\dagger}\hat{a}\rangle = 3sinh^2r. \quad (50)$$

Thus Eq. (50) is the sum of the mean photon number of the three SMSVSs.

In the same procedure, we can obtain

$$\langle\hat{a}^{\dagger}\rangle = \langle\hat{a}\rangle = 0, \quad (51)$$

$$\langle\hat{a}^{\dagger 2}\rangle = \langle\hat{a}^2\rangle = -3\sinh r\cosh r, \quad (52)$$

$$\langle(\hat{a}^{\dagger}\hat{a})^2\rangle = 12sinh^2r + 27sinh^4r. \quad (53)$$

The photon-number variance is defined as

$$(\Delta n)^2 = \langle(\hat{a}^{\dagger}\hat{a})^2\rangle - \langle\hat{a}^{\dagger}\hat{a}\rangle^2. \quad (54)$$

Employing Eqs. (50) and (53) into Eq. (54), results

$$(\Delta n)^2 = 12sinh^2r + 18\, sinh^4r. \quad (55)$$

we found the sum of the photon number variance of the three single mode vacuum states is grater than that of the superposition of the three single mode squeezed states. The photon-number variance of the three identical SMSVS superposed here is shown. The crushing properties of the superposed three identical SMSVS are depicted by a marginally changed definition of Eq. (27) as

$$(\Delta d \pm)^2 = 3 + \langle: \hat{d}\pm, \hat{d}\pm :\rangle. \quad (56)$$

Employing Eqs. (28) and (29) into Eq. (56), we obtain

$$(\Delta d \pm)^2 = 3 + 2\langle\hat{a}^{\dagger}\hat{a}\rangle \pm \langle\hat{a}^2\rangle \pm \langle\hat{a}^{\dagger 2}\rangle \mp \langle\hat{a}^{\dagger}\rangle^2 \mp \langle\hat{a}\rangle^2 - 2\langle\hat{a}^{\dagger}\rangle\langle\hat{a}\rangle \quad (57)$$

Substituting Eqs. (49-51) into, Eq. (56), yields

$$(\Delta d \pm)^2 = 3e^{\mp 2r}. \quad (58)$$

The last condition is the quadrature fluctuation of the superposed three indistinguishable SMSVSs. Comparatively to Eqs. (33), Eq. (58) is the amount of the quadrature change of the singular light pillars (Daoud, 2003). Typically, the quadrature pressing of the three superposed light shafts is calculated by comparing it to the quadrature fluctuation of a single transparent light bar. But it looks like this isn't the right way to do things currently(Zayed et al., 2005; Abdalla, 1994). Next, we contend that the quadrature squeezing for the superposed light beams and the quadrature variance of the three superposed coherent light beams need to be considered when computing the quadrature squeezing. It is certainly possible to determine the quadrature difference of three superposed reasonable light bars to be three by using Eq. (57). Eq. (58) shows that the pressing of the superposed states occurs in addition to quadrature. The last equation is the quadrature variance of the superposed three identical SMSVSs defined as

$$s = \frac{3-(\Delta d+)^2}{3} \quad (59)$$

and in view of Eq. (58), we see that

$$s = 1 - e^{-2r} \quad (35)$$

This is just the quadrature squeezing of the SMSVS.

CONCLUSIONS

The statistical and squeezing characteristics of an SMSVS using the Q function have been covered in this article. It was discovered that the average photon count rises as the squeezing parameter increases, reaching a dramatic peak for high values of r. Furthermore, we have observed that a single-mode squeezed vacuum state has super-Poisson photon statistics. Additionally, it has been demonstrated that the quadrature squeezing disappears at zero and increases to 100% as the squeeze parameter gets closer to infinity. Next, we applied the Q function of an SMSVS to derive the Q function for the superposed light beams. Next, using a modified definition of quadrature variance, we looked at the mean photon number, photon number variance, and quadrature squeezing. It has been observed that the quadrature variance and mean photon number rise when the three identical SMSVSs are superposed. Furthermore, we have discovered that the superposed light beams' photon statistics are super-Poisson and that their quadrature squeezing is identical to that of the single-mode squeezed vacuum light.

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DECLARATION

The authors declare that there is no conflict of interest.

DATA AVAILABILITY STATEMENT

All data are available from the corresponding author upon request.

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